Exercice 1 :

i) $f(x)=\sin\left(\ln(3x)\right).e^{x};D_{f}=\mathbb{R}_{+}^{*};f'(x)=\left[\frac{1}{x}\cos\left(\ln(3x)\right)+\sin\left(\ln(3x)\right)\right].e^{x}$

ii) $g(x)=\frac{e^{7x}}{\sqrt{2-x}};D_{g}=]-\infty,2[;g'(x)=\frac{7e^{7x}\sqrt{2-x}+\frac{e^{7x}}{2\sqrt{2-x}}}{2-x}$

iii) $h(x)=5^{2x}=e^{2x\ln 5};D_{h}=\mathbb{R};h'(x)=2\ln 5.h(x)$

Exercice 2 :

i) $\tan\left(\frac{\pi}{6}\right)=\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}=\frac{\sqrt{3}}{3}$ et $\tan\left(\frac{\pi}{4}\right)=1$.

ii) $\frac{1+\tan x}{1-\tan x}=\frac{\sqrt{3}}{3}=\tan\frac{\pi}{6}=\tan\left(\frac{\pi}{4}+x\right)\Rightarrow x=\frac{\pi}{6}-\frac{\pi}{4}+k\pi=-\frac{\pi}{12}+k\pi$.

Exercice 3 : f(x) = e^x.sin(x) = $x+x^{2}+\frac{x^{3}}{3}+o\left(x^{3}\right)$.

Exercice 4 :

i) $\lim_{x\to 0}\frac{1-\cos x}{\tan(x).\sin\left(3x\right)}=\frac{1}{6}$

ii) $\lim_{x\to 0}\frac{\ln\left(1-4x\right)}{1-e^{7x}}=\frac{4}{7}$

Exercice 5 :

i) $I=\int_{-1}^{1}\frac{\tan\left(x^{5}\right)}{\cos^{3}(x)\ln\left(x^{4}\right)}dx=0$

ii) $J=\int_{0}^{1}(5x-1)e^{2x}dx=\left[\left(\frac{5}{2}x-\frac{7}{4}\right)e^{2x}\right]_{0}^{1}=\frac{3e^{2}+7}{4}$

Exercice 6 : A(1, -2, 1), B(-1, 0, 2) et C(2,-3, 2).

i) $\overrightarrow{AB}.\overrightarrow{AC} = -3$ et $\overrightarrow{AB} \wedge \overrightarrow{AC} \begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix}$.

ii) P : $\begin{vmatrix} x-1 & -2 & 1 \\ y+2 & 2 & -1 \\ z-1 & 1 & 1 \end{vmatrix} = 0 \Leftrightarrow x + y + 1 = 0$.

iii) Q : $\overrightarrow{AM}.\overrightarrow{BC} = 0 \Leftrightarrow x - y = 3$.

iv) $\overrightarrow{AB}, \overrightarrow{AC}, \vec{u}$ coplanaires $\Leftrightarrow \begin{vmatrix} a & -2 & 1 \\ 1 & 2 & -1 \\ 1 & 1 & 1 \end{vmatrix} = 0 \Leftrightarrow a = -1$.

Exercice 7 :

$i)$ $f(x,y) = e^{\left(x^2+y^2\right)} \Rightarrow \left(\dfrac{\partial f}{\partial x}(x,y) = 2xe^{\left(x^2+y^2\right)} ; \dfrac{\partial f}{\partial y}(x,y) = 2ye^{\left(x^2+y^2\right)} \right)$

$\Rightarrow \left(\dfrac{\partial f}{\partial x} \text{ et } \dfrac{\partial f}{\partial y} \text{ continues sur } \mathbb{R}^2 \right) \Rightarrow \left(df(x,y) = 2xe^{\left(x^2+y^2\right)}dx + 2ye^{\left(x^2+y^2\right)}dy ; \forall (x,y) \in \mathbb{R}^2 \right)$

$ii)$ $g(x,y) = x^3 - 5x\ln(y) \Rightarrow \left(\dfrac{\partial f}{\partial x}(x,y) = 3x^2 - 5\ln(y) ; \dfrac{\partial f}{\partial y}(x,y) = \dfrac{-5x}{y} \right)$

$\Rightarrow \left(\dfrac{\partial f}{\partial x} \text{ et } \dfrac{\partial f}{\partial y} \text{ continues sur } \mathbb{R} \times \mathbb{R}_+^* \right) \Rightarrow \left(df(x,y) = 3x^2 - 5\ln(y)\,dx + \dfrac{-5x}{y}dy ; \forall (x,y) \in \mathbb{R} \times \mathbb{R}_+^* \right)$

Exercice 8 :

Soit $\vec{V} : \left| \begin{array}{l} \mathbb{R}^3 \quad \rightarrow \quad \mathbb{R}^3 \\ M(x,y,z) \mapsto \vec{V}(M) \begin{pmatrix} ye^{xy} + \dfrac{z}{x} + yz \\ -\cos z + xe^{xy} + xz \\ \ln x + xy + y\sin z \end{pmatrix} \end{array} \right.$, pour $x > 0$.

i) $\vec{V}$ dérive du potentiel scalaire f / f(M) = xyz – y cos z + z ln | x | + e^{xy} + c ; c$\in \mathbb{R}$.

ii) $div_M \vec{V} = y^2e^{xy} - \dfrac{z}{x^2} + x^2e^{xy} + y\cos z$ et $\overrightarrow{rot}_M \vec{V} = \vec{0}$.