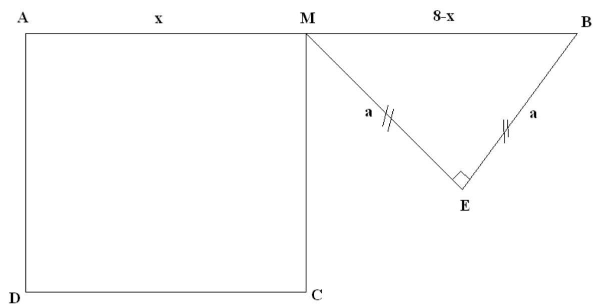
Correction

Exo 1 :

1- Pythagore : $2a^2 = (8 - x)^2$, donc $f(x) = x^2 + \frac{(8-x)^2}{4} = \frac{5}{4}x^2 - 4x + 16$.



2- $f'(x) = \frac{5}{2}x - 4$, d'où le tableau de variations :

x	0		8/5		8
f '(x)		—	0	+	
f	16	↘	64/5	↗	64

Donc : Max = $64 = f(8)$ et Min = $\frac{64}{5} = f\left(\frac{8}{5}\right)$.

Exo 2 :

1- $z_1 = \sqrt{2}e^{i\frac{\pi}{4}}$ et $z_2 = 2e^{-i\frac{\pi}{6}}$.

2- $(z_1.z_2) = (\sqrt{3}+1) + i(\sqrt{3}-1) = 2\sqrt{2}e^{i\frac{\pi}{12}}$.

3- $\cos\left(\frac{\pi}{12}\right) = \frac{\sqrt{3}+1}{2\sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{4}$ et $\sin\left(\frac{\pi}{12}\right) = \frac{\sqrt{6}-\sqrt{2}}{4}$.

Exo 3 :

1- $\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0}\left(x\ln(x+2) - x\ln(x)\right) = 0 - 0 = 0$.

2- Non : $\lim\limits_{x\to 0} \dfrac{f(x)-f(0)}{x} = \lim\limits_{x\to 0} \ln\left(\dfrac{x+2}{x}\right) = +\infty$.

3- $f(x) = x\ln(1+h(x))$.

4- $\lim\limits_{x\to +\infty} f(x) = \lim\limits_{x\to +\infty}\left(xh(x)\right) = \lim\limits_{x\to +\infty}\left(\dfrac{2x}{x}\right) = 2$.

5- $f'(x) = \ln\left(x+2\right)x - \dfrac{2}{x+2}$ et $f''(x) = -\dfrac{2}{x(x+2)} + \dfrac{2}{(x+2)^2} = \dfrac{-4}{x(x+2)^2}$.

6- Variations de f' :

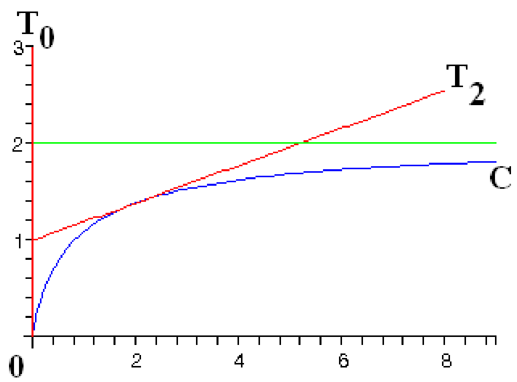
x	0		+∝
f ''(x)		—	
f '	+∝	↘	0

7- Variations de f :

x	0		+∝
f ' (x)		+	
f	0	↗	2

8- T_0 : x = 0 et T_2 : $y = \left(\ln(2) - \dfrac{1}{2}\right)(x-2) + 2\ln(2)$.

9- Graphe :



Exo 4 :

1- $u_{n+1} = u_n$.
2- $u_n = a_0 + b_0 = 8$.
3- $v_{n+1} = \dfrac{1}{5} v_n$.
4- $v_n = \left(\dfrac{1}{5}\right)^n (a_0 - b_0) = 2\left(\dfrac{1}{5}\right)^n$.
5- Oui : vers 0.
6- $a_n = \dfrac{u_n + v_n}{2} = 4 + \left(\dfrac{1}{5}\right)^n$ et $b_n = \dfrac{u_n - v_n}{2} = 4 - \left(\dfrac{1}{5}\right)^n$.
7- $\lim\limits_{n \to +\infty} a_n = \lim\limits_{n \to +\infty} b_n = \dfrac{a_0 + b_0}{2} = 4$.